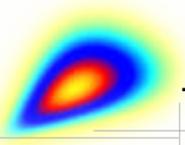


The Iman-Conover Method

Stephen Mildenhall
Aon Re Services
CAS Spring Meeting
May 17, 2004

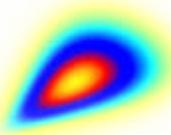


The Problem

- ▶ Estimate Bivariate Distribution of Ceded and Retained Losses
 - ▶▶ Excess of Loss Reinsurance
 - ▶▶ Profit or Sliding Scale Commission

- ▶ Why?
 - ▶▶ Distribution of Net Underwriting Result

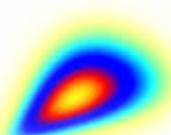
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The Problem

- ▶ Estimate Univariate Distribution of Gross Losses
- ▶ Determine Univariate Ceded and Retained Losses
 - ▶ FFT Vs. Method of Moments
- ▶ Determine Bivariate Distribution
 - ▶ Bivariate FFT Vs. Iman Conover
 - ▶ Copulas

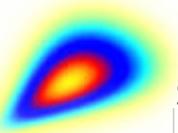
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The Details

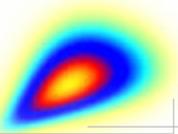
- ▶ Complete, Detailed Explanation in my Chapter
 - ▶ All Formulas
 - ▶ References
 - ▶ Step-by-Step Instructions

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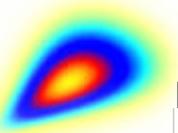
Setup

- ▶ Severity: Lognormal $\mu=9.0$, $\sigma=2.0$
- ▶ Policy Limit \$1M
 - ▶ Average Severity \$47,429
- ▶ Gross Expected Losses \$25M
 - ▶ Expected Claim Count 527
- ▶ Excess Reinsurance \$800,000 xs \$200,000
 - ▶ Retained Severity \$31,591
 - ▶ Excess Conditional Severity \$290,985
 - ▶ Probability of Attaching 5.446%, 28.7 Expected Claims
- ▶ Hogg and Klugman or Klugman, Panjer and Wilmot



Frequency Assumption

- ▶ Negative Binomial With Meyers Contagion c
- ▶ Variance of Contagion Distribution $c = 0.0625$
 - ▶ Implies Asymptotic CV of Aggregate 25%



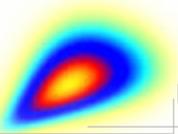
Moments

Severity	Ground Up	Retained	Ceded
EX	47,439.0	31,591.0	290,985.3
CV	2.722	1.675	0.951
Skewness	5.238	2.235	0.837
Frequency			
EN	527.0	527.0	28.7
CV	0.254	0.254	0.312
Skewness	0.500	0.500	0.512
Aggregate			
EA	25,000,000.0	16,648,209.8	8,351,790.2
CV	0.280	0.264	0.359
Skewness	0.513	0.502	0.554

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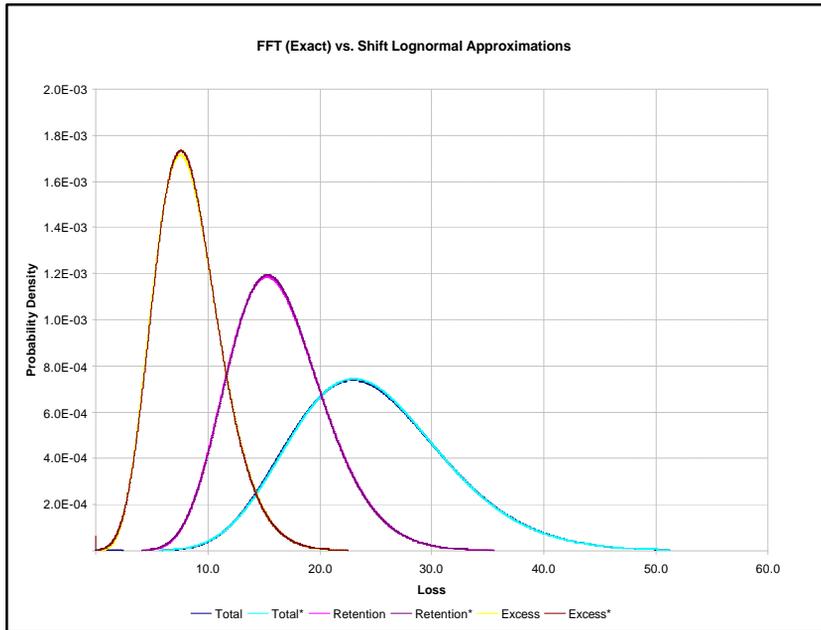
Univariate Distributions

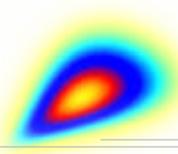
- ▶ Fast Fourier Transform
 - ▶▶ Essentially Exact
 - ▶▶ Very Fast
 - ▶▶ Some Programming Required
- ▶ Method of Moments
 - ▶▶ Match Mean and CV to Lognormal
 - ◆ Poor Fit, Typically Too Skewed
 - ▶▶ Match Mean, CV and Skewness to Shifted Lognormal
 - ◆ Stellar Fit in Almost All Situations

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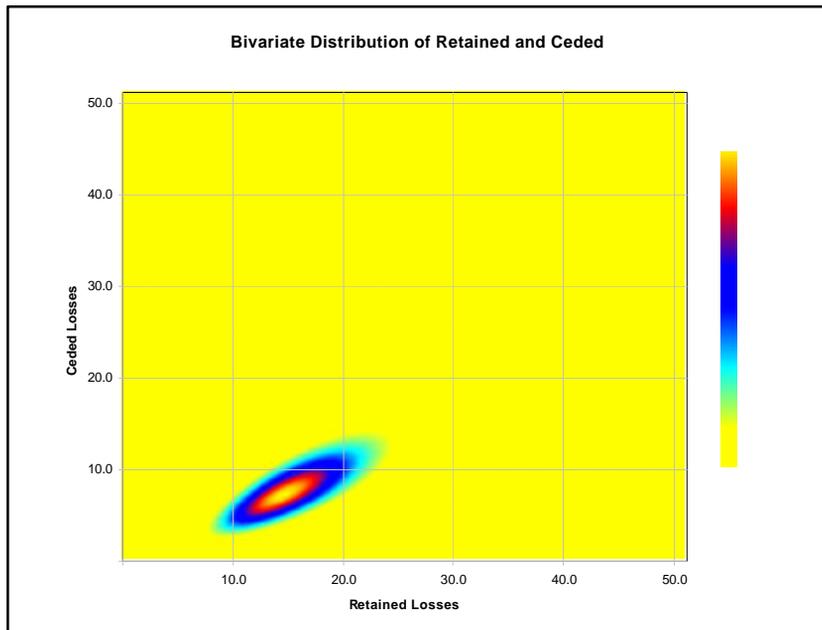


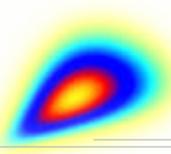


Joint Distribution

- ▶ Two Dimensional FFT:
- ▶ $(A,B)=(R_1,C_1)+\dots+(R_N,C_N)$
 - ▶ A=Aggregate Retained Losses
 - ▶ B=Aggregate Ceded Losses
 - ▶ R=Retained Loss Severity Distribution
 - ▶ C=Ceded Loss Severity Distribution
 - ▶ N=Claim Count Distribution
 - ▶ Apply Same Method as For Univariate FFT
- ▶ Know Joint Distribution of (R,C)
 - ▶ Supported Along x-Axis to \$200,000
 - ▶ Vertically Along (200000,0) to (200000,800000)
 - ▶ Correlation Between R and S is 0.182

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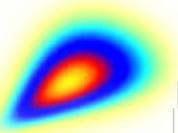




Iman Conover Method

- ▶ Need Alternatives to FFT Method
- ▶ Iman Conover (IC) Method in a Nutshell
 - ▶ Given Input Sample from Marginal Distributions
 - Re-order to Have Same RANK ORDER as a Reference Distribution With Desired Linear Correlation
- ▶ Method Effective Because
 - ▶ Rank and Linear Correlation Close
 - ▶ Easy to Produce Reference Distributions
- ▶ IC Used By At Risk

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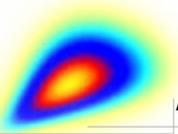
Iman Conover Method

- ▶ Inputs
 - ▶▶ Sample ($n \times r$ matrix) From Marginal Distributions
 - ◆ $n \sim 10,000$, $r=2$ for Bivariate Distribution
 - ▶▶ Correlation Matrix ($r \times r$ matrix)
- ▶ Output
 - ▶▶ SamPle Re-ordered to Have the Same Rank Correlation As a Reference Distribution With Desired Linear Correlation
- ▶ Reference Distributions Use Choleski Trick

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The Iman Conover Method

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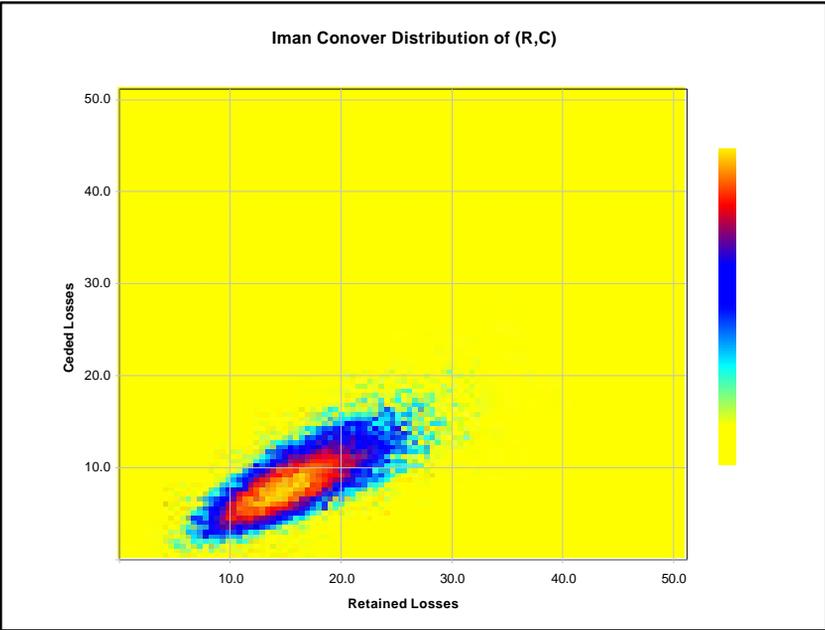
Applying Iman Conover Method

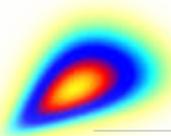
- ▶ Need Correlation Between Retained Losses A and Ceded Losses B
- ▶ $\text{Cov}(A,B) = cn^2E(R)E(C) + naE(C)$
 - ▶▶ $c=0.0625$, Frequency Contagion Parameter
 - ▶▶ $n=527$, Ground Up Expected Claim Count
 - ▶▶ $a=200,000$, Attachment
 - ▶▶ $E(C)$ is Ground-Up Severity, \$15,848
- ▶ Correlation Driven by Common Frequency
 - ▶▶ Correlation=0.786

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The Iman Conover Method

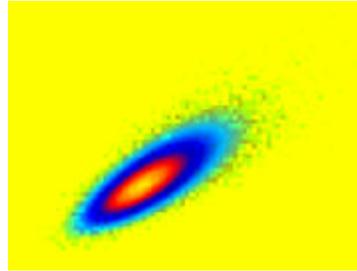
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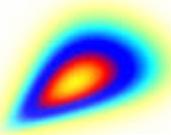


Iman Conover Solution

- ▶ Laying IC Over FFT Shows Two Plots Agree
 - ▶ FFT Exact Density
 - ▶ IC Simulated Sample Distribution

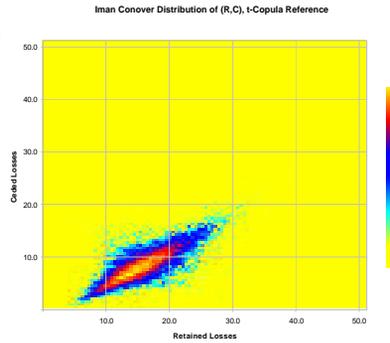


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Alternative Iman Conover Solutions

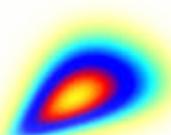
- ▶ Choice: Reference Distribution
- ▶ Default: Normal Copula
 - ▶▶ At Risk Uses Normal Copula
- ▶ Any Reference Will Do!
- ▶ Chart Shows t-Copula
 - ▶▶ Pinched Look
 - ▶▶ Tail Dependence



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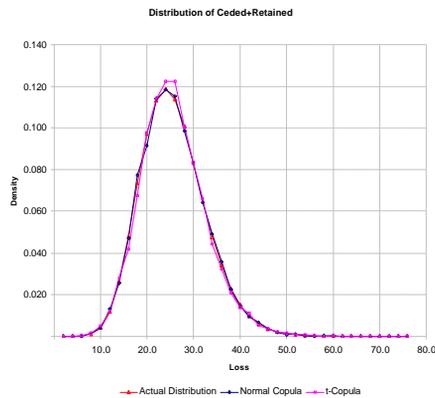
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Alternative Iman Conover Solutions

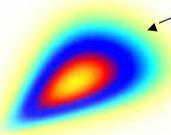
- ▶ Know Distribution of Sum
 - ▶▶ Gross Losses
- ▶ Normal Copula Closer to Actual
- ▶ Normal Copula is "Minimal Entropy" Solution
 - ▶▶ Fewest Assumptions
- ▶ Normal Copula Not Appropriate For Other Processes
 - ▶▶ FX, Stock Prices



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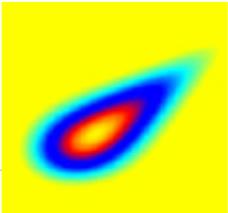
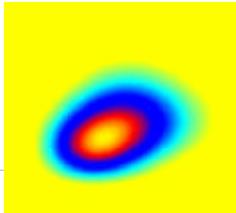
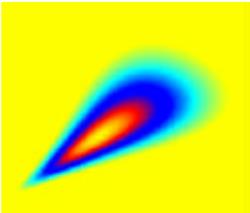


Clayton

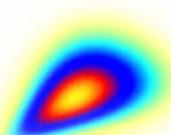
Copulas

- ▶ Copulas Determine Association in Multivariate Distributions
- ▶ Many Copulas Available
- ▶ Can Use Copula Methods as Alternative

Cook FGM Venter HRT



The slide displays four density plots of different copulas. The top plot is labeled 'Clayton' and shows a distribution concentrated in the lower-left corner. Below it are three smaller plots: 'Cook' (lower-left concentration), 'FGM' (circular, symmetric concentration), and 'Venter HRT' (diagonal concentration).



Resources

- ▶ Presentation and Software Available At
 - ▶ <http://www.mynl.com/wp>
- ▶ Software
 - ▶ FFT and Copula Methods
 - ▶ Iman Conover Implementation
 - ▶ Graphics
 - ▶ General Actuarial Methods

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